I.1.1- $\vec{\Omega} = \Omega\cos\lambda\,\vec{u_y} + \Omega\sin\lambda\,\vec{u_z}$

$$\vec{F_{ic}} = -2m \begin{vmatrix} 0 \\ \Omega\cos\lambda \\ +\Omega\sin\lambda \end{vmatrix} \wedge \begin{vmatrix} \dot x \\ \dot y \\ \dot z \end{vmatrix} = -2m\Omega \begin{vmatrix} -\sin\lambda\,\dot y + \cos\lambda\,\dot z \\ \sin\lambda\,\dot x \\ -\cos\lambda\,\dot x \end{vmatrix}$$

I.1.2- $\varepsilon = \frac{\|\vec{F_{ic}}\|}{mg} \leq \frac{2\Omega}{g} v$

$\Rightarrow \varepsilon \leq 2\Omega\sqrt{\frac{l}{g}}$

ordre de grandeur $v = l\omega_0 = l\sqrt{\frac{g}{l}} = \sqrt{lg}$

$\varepsilon \leq 1{,}75\,10^{-4}$

$\|\vec{F_{ic}}\| \ll mg$

I.1.3. La force d'inertie d'entraînement est inclue dans l'expression du poids

I.1.4 Le mouvement de M se fait au voisinage du plan xy

$z \simeq 0$ $\quad \dot z = \ddot z = 0$

$$m \begin{vmatrix} \ddot x \\ \ddot y \\ \ddot z \end{vmatrix} = \begin{vmatrix} 0 \\ 0 \\ -mg \end{vmatrix} + \frac{T}{l} \begin{vmatrix} -x \\ -y \\ h-z \end{vmatrix} - 2m\Omega \begin{vmatrix} \cos\lambda\,\dot z - \sin\lambda\,\dot y \\ \sin\lambda\,\dot x \\ -\cos\lambda\,\dot x \end{vmatrix}$$

$$m\ddot x = -\frac{T}{l}x + 2m\Omega\sin\lambda\,\dot y$$

$$m\ddot y = -\frac{T}{l}y - 2m\Omega\sin\lambda\,\dot x$$

$$0 \simeq -mg + \frac{T}{l}h \qquad (l \simeq h)$$

$$\Rightarrow mg \simeq T$$

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$$\begin{cases} \ddot{x} - 2\Omega \sin\lambda \dot{y} + \frac{g}{\ell} x = 0 & ① \\ \ddot{y} + 2\Omega \sin\lambda \dot{x} + \frac{g}{\ell} y & ② \end{cases}$$

11.5. $\varepsilon' = \frac{\Omega}{\omega_0} = 8{,}75\,10^{-4}$ $\quad \Omega \ll \omega_0$

11.6. $① + j②$ $\Rightarrow$ $\ddot{\underline{u}} + 2j\Omega' \dot{\underline{u}} + \omega_0^2 \underline{u} = 0$

éq caractéristique $r^2 + 2j\Omega' r + \omega_0^2$

$r = -j\Omega' \pm j\sqrt{\Omega'^2 + \omega_0^2} = -j\Omega' \pm j\omega$

$\underline{u} = e^{-j\Omega' t}\left(\underline{\alpha} e^{j\omega t} + \underline{\beta} e^{-j\omega t}\right)$; $\omega = \sqrt{\Omega'^2 + \omega_0^2} \simeq \omega_0$

11.7) $\underline{u}(t=0) = \alpha + \beta = x_0$

~~$\dot{\underline{u}}(t=0)$ ... $(\alpha + \beta)$ ...~~

$\dot{\underline{u}}(t=0) = 0 = -j\Omega'(\alpha + \beta) + j\omega(\alpha - \beta)$

$\alpha = \frac{\omega + \Omega'}{2\omega} x_0$ $\quad$ $\beta = \frac{\omega - \Omega'}{2\omega} x_0$

$\underline{u} = \left[x_0 \cos\omega t + j\frac{\Omega'}{\omega} x_0 \sin\omega t\right] e^{-j\Omega' t}$

$x(t) = x_0\left[\cos\omega t \cos\Omega' t + \frac{\Omega'}{\omega} \sin\omega t \sin\Omega' t\right]$

$y(t) = x_0\left[-\cos\omega t \sin\Omega' t + \frac{\Omega'}{\omega} \sin\omega t \cos\Omega' t\right]$

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$\Omega' t \ll 1$ pour une durée $t \ll T \simeq 24\,h$

$\sin \Omega' t \simeq 0$

$\cos \Omega' t = 0$ $\Rightarrow$ $x = x_0 \cos \omega t$

$y = x_0 \frac{\Omega'}{\omega} \sin \omega t$

$$\left(\frac{x}{x_0}\right)^2 + \left(\frac{y}{x_0 \frac{\Omega'}{\omega}}\right)^2 = 1$$ équation d'une ellipse

I.18.1) $\tau = \frac{3T}{2}$

I.18.2) $t_1 = \frac{T}{2}$ et $t_2 = T$

$t = T$, $\omega T = 2\pi$ et $\Omega' T = \frac{2\pi \Omega'}{\omega} \ll 1$

$$x = x_0 \cos\left(\frac{2\pi \Omega'}{\omega}\right) \simeq x_0 \left(1 - \frac{1}{2}\left(\frac{2\pi \Omega'}{\omega}\right)^2\right)$$

$$y = -x_0 \sin\left(\frac{2\pi \Omega'}{\omega}\right) \simeq -\frac{2\pi \Omega'}{\omega} x_0$$

$$\tan \varphi = \frac{y}{x_0} \simeq \varphi = -\frac{2\pi \Omega'}{\omega} \Rightarrow \boxed{\varphi = -\frac{2\pi \Omega'}{\omega}}$$

I.18.3) $|\varphi| = \omega_F T = \frac{2\pi |\Omega'|}{\omega}$

$$T_F = \frac{2\pi}{\omega_F} = \frac{2\pi T \omega}{2\pi |\Omega'|} = \frac{2\pi}{|\Omega'|} \Rightarrow \boxed{T_F = \frac{2\pi}{\Omega |\sin \lambda|}}$$

~~Dans l'hémisphère Nord~~

si $\sin \lambda > 0$ $\varphi < 0$ sens ~~d'oscillation~~

~~sens~~ sens l'aiguille d'une montre

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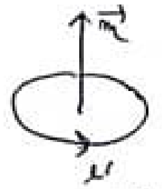
II-1-



$\vec{m} = i'\pi r'^2 \vec{e_z}$

II.1.2 . $\vec{B} = \vec{rot}\,\vec{A} = \frac{\mu_0 \vec{rot}}{4\pi}\left(\frac{\vec{m}\wedge\vec{r}}{r^3}\right) = \frac{\mu_0}{4\pi}\vec{rot}\left(\frac{\vec{m}\wedge\vec{r}}{r^3}\right)$

$\vec{rot}\left(\frac{\vec{m}\wedge\vec{r}}{r^3}\right) = \frac{1}{r^3}\vec{rot}(\vec{m}\wedge\vec{r}) + \vec{grad}\left(\frac{1}{r^3}\right)\wedge(\vec{m}\wedge\vec{r})$

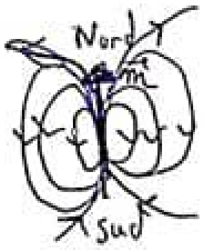
$\vec{rot}(\vec{m}\wedge\vec{r}) = \vec{\nabla}\wedge(\vec{m}\wedge\vec{r}) = \begin{vmatrix}\frac{\partial}{\partial x}\\ \frac{\partial}{\partial y}\\ \frac{\partial}{\partial z}\end{vmatrix} \wedge \left(\begin{vmatrix} m_x \\ m_y \\ m_z\end{vmatrix} \wedge \begin{vmatrix} x \\ y \\ z\end{vmatrix}\right) = \begin{vmatrix}\frac{\partial}{\partial x}\\ \frac{\partial}{\partial y}\\ \frac{\partial}{\partial z}\end{vmatrix} \wedge \begin{vmatrix} y m_z - z m_y \\ x m_z - z m_x \\ y m_x - x m_y\end{vmatrix}$

$\vec{rot}(\vec{m}\wedge\vec{r}) = \begin{vmatrix} 2m_x \\ 2m_y \\ 2m_z\end{vmatrix} = 2\vec{m}$

$\vec{grad}\left(\frac{1}{r^3}\right)\wedge(\vec{m}\wedge\vec{r}) = -\frac{3\vec{r}}{r^5}\wedge(\vec{m}\wedge\vec{r}) = -\frac{3\vec{m}\,r^2}{r^5} + 3(\vec{m}.\vec{r})\vec{r}$

$\vec{B} = \frac{\mu_0}{4\pi r^5}\left[3(\vec{m}.\vec{r})\vec{r} - r^2\vec{m}\right]$

II-1.3



II.1.4)

Le plan $(P\vec{u_r}\vec{u_z})$ plan d'antisymétrie de l'aimant (aimant distribution de boucle de courants) $\Rightarrow \vec{B} \in (P\vec{u_r}\vec{u_z})$, $B_\theta = 0$

$\vec{r} = \vec{MP} = \rho\vec{u_\rho} + (z_0 - z)\vec{u_z}$

$\vec{B} = \frac{\mu_0 m}{4\pi r^5}\left[3(z_0-z)(\rho\vec{u_\rho} + (z_0-z)\vec{u_z}) - r^2 \vec{u_z}\right]$

$\vec{B} = \frac{\mu_0 m}{4\pi r^5}\left[3\rho(z_0-z)\vec{u_\rho} + 3(z_0-z)^2 - \rho^2 - (z_0-z)^2\right]$

$\vec{B} = \underbrace{\beta\frac{3\rho(z_0-z)}{(\rho^2+(z_0-z)^2)^{\frac{5}{2}}}}_{B_\rho}\vec{u_\rho} + \underbrace{\beta\frac{2(z_0-z)^2-\rho^2}{(\rho^2+(z_0-z)^2)^{\frac{5}{2}}}}_{B_z}\vec{u_z}$

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/ II 2. II - 2.1

$$\vec{E}(P) = -\frac{\partial \vec{A}}{\partial t} = -\frac{\partial \vec{A}}{\partial z} \times \frac{\partial z}{\partial t} = -V\frac{\partial \vec{A}}{\partial z}$$ (cas Neumann)

$$\vec{A} = \beta \frac{\vec{u}_z \wedge (\rho \vec{u}_\rho + (z_0 - z)\vec{u}_z)}{(\rho^2 + (z_0-z)^2)^{3/2}} \Rightarrow \vec{A} = \frac{\beta \rho}{(\rho^2 + (z_0-z)^2)^{3/2}} \vec{u}_\theta$$

$$\frac{\partial \vec{A}}{\partial z} = \frac{\beta\rho \times \left(-\frac{3}{2}\right) \times -2(z_0 - z)}{(\rho^2 + (z_0-z)^2)^{\frac{5}{2}}} \vec{u}_\theta = \frac{3\beta\rho(z_0-z)}{(\rho^2 + (z_0-z)^2)^{5/2}} \vec{u}_\theta$$

$$\vec{E}(P) = -\frac{3\beta\rho(z_0 - z)}{(\rho^2 + (z_0-z)^2)^{\frac{5}{2}}} V \vec{u}_\theta$$

ou $\vec{E}_P = -\vec{V} \wedge \vec{B} = -V\vec{u}_z \wedge B_\rho \vec{u}_\rho \Rightarrow E_P = -V B_\rho \vec{u}_\theta = -\frac{3\beta\rho(z_0 - z)V}{(\rho^2 + (z_0-z)^2)^{\frac{5}{2}}} \vec{u}_\theta$

(cas Lorentz)

II-2.1

$$e_s = \oint_{spire} \vec{E}_P \cdot d\vec{l} = \oint_{spire} E_P \rho \, d\theta = \rho E_P \times 2\pi$$

$$e_s = -\frac{6\pi\beta\rho^2(z_0 - z)}{(\rho^2 + (z_0-z)^2)^{5/2}} V = -2\pi\beta g \rho V \qquad \boxed{e_s = -2\pi\beta\rho g V}$$

II-2.2)

$$i = \frac{e_s}{r_s} = \frac{N e_s}{R}$$

II.2.3

$$\vec{F}_L = i \oint_{spire} d\vec{l} \wedge \vec{B} = i \oint_{spire} \rho \, d\theta \, \vec{u}_\theta \wedge (\beta g \vec{u}_\rho + \beta h \vec{u}_z)$$

$$= -2\pi i \rho \beta g \vec{u}_z + \underbrace{i\rho\beta h \int_0^{2\pi} \vec{u}_\rho \, d\theta}_{0}$$

$$\boxed{\vec{F}_L = -2\pi i \rho \beta g \vec{u}_z} \qquad \boxed{\vec{F}_L = -2\pi i \rho \beta g \vec{u}_z}$$

$$\vec{F}_L = \frac{36\pi^2 N \rho^4}{R} V \frac{(z_0-z)^2}{(\rho^2 + (z_0-z)^2)^5} \vec{u}_z$$

si $V > 0$: $\vec{F}_L$ est dirigée selon $\vec{u}_z$, la spire s'éloigne de l'aimant

si $V < 0$: $\vec{F}_L$ est dirigée selon $-\vec{u}_z$ la spire s'approche de l'aimant, Loi de Lenz est vérifiée

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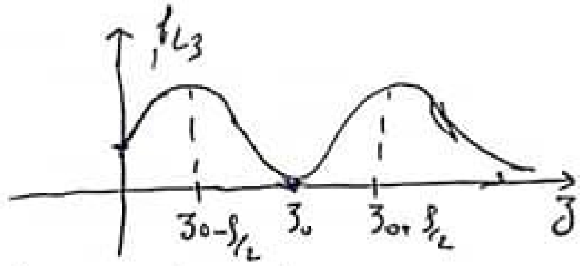
II-2-4 $b_{Lz} = \frac{36\pi^2 N\beta\rho^4 v}{R}\,\frac{(z_0-z)^2}{(\rho^2+(z_0-z)^2)^5}$, $b_{Lz} = A\frac{x^2}{(\rho^2+x^2)^5}$

$$\frac{db_{Lz}}{dx}\Big| = 0 = \frac{2x(\rho^2+x^2)^5 - 10x^3(\rho^2+x^2)^4}{(\rho^2+x^2)^{10}} \Rightarrow 2x(\rho^2+x^2)^4(\rho^2+x^2-5x^2)=0$$

$x=0$, $x=\pm\frac{\rho}{2}$



II-3 . La f.e.m induite dans la bobine élémentaire d'épaisseur dz est

$$de = dN e_s = n e_s\, dz$$

$$e = n\int_{z_0-\frac{\ell}{2}}^{z_0+\frac{\ell}{2}} e_s\, dz \Rightarrow e = -6\pi\beta\rho^2 v n \underbrace{\int_{z_0-\ell/2}^{z_0+\ell/2}\frac{(z_0-z)\,dz}{(\rho^2+(z_0-z)^2)^{5/2}}}_{J}$$

$$e = -\frac{6\pi\beta N\rho^2 v}{\ell}J = -\gamma v$$

$$\boxed{\gamma = \frac{6\pi\beta N\rho^2}{\ell}J}$$

II.3-2

$$dF_{Lz} = n\, b_{Lz}\, dz \Rightarrow F_{Lz} = +\frac{36\pi^2N^2\rho^4 v}{\ell R}\underbrace{\int_{z_0-\ell/2}^{z_0+\frac{\ell}{2}}\frac{(z_0-z)^2}{(\rho^2+(z_0-z)^2)^{5/2}}}_{J'}$$

$$F_{Lz} = \delta v$$

$$\boxed{\delta = \frac{36\pi^2N^2\rho^4}{\ell R}J'}$$

II-4-1- $i = \frac{e_1-e_2}{2R}$

II-4-2- $e_1 = -\gamma v_1 = -\gamma\dot z_1$ et $e_2 = +\gamma v_2 = +\gamma\dot z_2$

$$\boxed{i = -\frac{\gamma}{2R}(\dot z_2+\dot z_1)}$$

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II.4.3 $F_{12} = -\frac{\delta}{2}(\dot{z}_2 + \dot{z}_1)$ (on remplace V par $\dot{z}_2 - \dot{z}_1$ et R par $2R$)

$F_{22} = -\frac{\delta}{2}(\dot{z}_2 + \dot{z}_1)$

II-5-1 <u>PFD appliqué à M</u>

$$\cancel{M}\ddot{z}_1 = Mg - k\Delta l(t) + F_{12} = -kz_1 - \frac{\delta}{2}(\dot{z}_2 - \dot{z}_1)$$

$$\cancel{\ddot{z}_1 + \frac{\delta}{2M}\dot{z}_1 + \frac{k}{M}z_1 = 0} \Rightarrow$$

$$\boxed{\ddot{z}_1 + \lambda(\dot{z}_2 + \dot{z}_1) + \omega^2 z_1 = 0} \quad 1$$

de m̂

$$\boxed{\ddot{z}_2 + \lambda(\dot{z}_2 + \dot{z}_1) + \omega^2 z_2 = 0}$$

II.5.2 $\cancel{\ddot{z} + 2\lambda\dot{z}}$ $\ddot{z} + 2\lambda(\dot{z}_2 - \dot{z}_1) + \omega^2 z = 0$

$$\boxed{\begin{cases} \ddot{z} + 2\lambda\dot{z} + \omega^2 z = 0 \\ \ddot{z}' + \omega^2 z' = 0 \end{cases}}$$

$\boxed{z' = z_{10}\cos\omega t}$ et $\boxed{z(t) = e^{-\lambda t}\left(z_{10}\cos\omega' t + \frac{\lambda}{\omega} z_{10}\sin\omega' t\right)}$

$\boxed{\omega' = \sqrt{\omega^2 - \lambda^2}}$

$z'(t) = a\cos\omega t + b\sin\omega t \Rightarrow z'(t=0) = z_{10} = a$

$\dot{z}'(t=0) = 0 = b\omega \Rightarrow b = 0$ $\Rightarrow \boxed{z'(t) = z_{10}\cos\omega t}$

<u>solution de $z(t)$</u>

Equation caractéristique $r^2 + 2\lambda r + \omega^2 = 0$

$r = -\lambda \pm j\sqrt{\omega^2 - \lambda^2} = -\lambda \pm j\omega'$ $(\omega' = \sqrt{\omega^2 - \lambda^2})$

$z(t) = e^{-\lambda t}(a' e^{-j\omega' t} + b' e^{+j\omega' t})$ $\cancel{= e^{-\lambda t}[a'\cos\omega' t + b'\sin\omega' t]}$

$\cancel{z(t=0) = c + d = z_{10}}$

$\cancel{\dot{z}(t=0) = -\lambda(c+d) + j\omega'(d-c) = 0 \Rightarrow (\lambda + j\omega')c = (-\lambda + j\omega')d}$

$\cancel{z(t) = e^{-\lambda t}}$ $z(t) = a' = z_{10}$

$\cancel{z(t=0) = a' + b' = 0}$ $z'(t) = (-\lambda'$

$\cancel{z'(t=0) = -\lambda(\ldots)}$

1) $\vec{v}(M/R) = l\dot{\theta}\vec{u_\theta}$, $\vec{a} = -l\dot{\theta}^2\vec{u_r} + l\ddot{\theta}\vec{u_\theta}$

2) $E_p = -mgx + cte = -mgl\cos\theta + cte$, $E_p(\theta) = mgl(1-\cos\theta)$

3) $E_M = \frac{1}{2}ml^2\dot{\theta}^2 + mgl(1-\cos\theta)$

4) $E_M = cte$ $\quad \frac{dE_M}{dt} = 0 = (ml^2\ddot{\theta} + mgl\sin\theta)\dot{\theta}$

$$\ddot{\theta} + \frac{g}{l}\sin\theta = 0$$

5) $\ddot{\theta} + \omega_0^2\theta = 0$, $\theta = \theta_0\cos\omega_0 t$, $T_0 = 2\pi\sqrt{\frac{l}{g}}$

6.1. $E_{M1} = \frac{1}{2}ml^2\dot{\theta}^2 + mgl(1-\cos\theta) = mgl(1-\cos\theta_0)$

$$\dot{\theta} = \pm\sqrt{2\frac{g}{l}(\cos\theta - \cos\theta_0)}$$

$$\int_0^{T/4} dt = \int_0^{\theta_0}\frac{d\theta}{\dot{\theta}} = \int_0^{\theta_0}\frac{d\theta}{\sqrt{2\frac{g}{l}(\cos\theta-\cos\theta_0)}}$$

$$T = 4 \times \sqrt{\frac{l}{2g}} \int_0^{\theta_0}\frac{d\theta}{\sqrt{2\frac{g}{l}(\cos\theta-\cos\theta_0)}}$$

$$T = T_0\frac{\sqrt{2}}{\pi}\int_0^{\theta_0}\frac{d\theta}{\sqrt{\cos\theta-\cos\theta_0}} = T_0 I(\theta_0)$$

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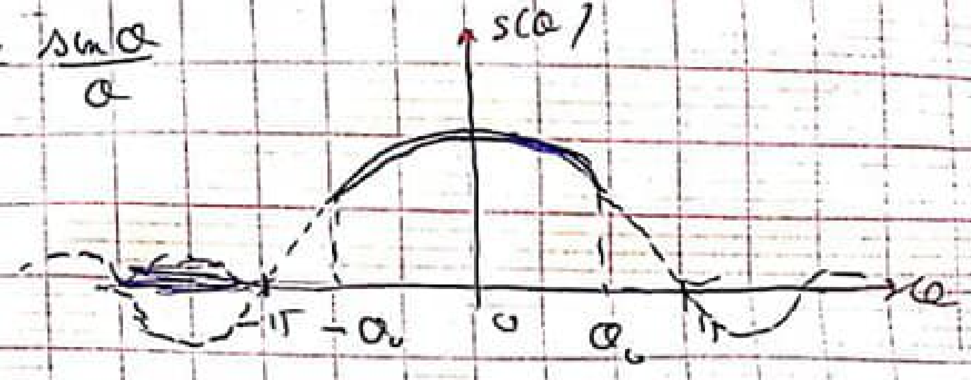
6-2 $\ddot{\theta} + \frac{g}{\ell} \frac{\sin\theta}{\theta}\, \theta = 0 \Rightarrow \ddot{\theta} + \frac{g}{\ell} S(\theta) \cdot \theta = 0$

$S(\theta) = \frac{\sin\theta}{\theta}$



6.3)

$T_1' = 2\pi \sqrt{\frac{\ell}{g_{eff}}} = 2\pi \sqrt{\frac{\ell}{g\, S(\theta_0)}} = T_0 \times S(\theta_0)^{-1/2}$

$S(\theta_0)^{-1/2} = \left(\frac{(\theta_0 - \theta_0^3/6)}{\theta_0} \right)^{-1/2} = \left(1 - \frac{\theta_0^2}{6}\right)^{-\frac{1}{2}}$

$= 1 + \frac{\theta_0^2}{12}$

$$\boxed{T_1' = T_0 \left(1 + \frac{\theta_0^2}{12}\right)}$$

7) $\|\vec{v}(\theta = \pi)\| \neq 0$ et $\|\vec{T}\|_{(\theta=\pi)} \neq 0$

8)

$$z(t) = e^{-\lambda t}\left[a' e^{-j\omega' t} + b' e^{j\omega' t}\right]$$

$$z(t=0) = a' + b' = z_{10}$$

$$\dot{z}(t=0) = -\lambda(a'+b') + j\omega'(b'-a') = 0 \Rightarrow (j\omega'+\lambda)a' = (j\omega'-\lambda)b'$$

$$a' = \frac{1}{2}\left(1 - \frac{\lambda}{j\omega}\right)z_{10} \; ; \; \cancel{b' = \frac{1}{2}\left(1 + \frac{\lambda}{j\omega'}\right)} z_{10} \quad b' = \frac{1}{2}\left(1 + \frac{\lambda}{j\omega}\right)z_{10}$$

$$z(t) = z_{10} e^{-\lambda t}\left[\cos\omega' t + \frac{\lambda}{\omega'}\sin\omega' t\right]$$

$$z_1 + z_2 = z'(t) = z_{10}\cos\omega t$$

$$z_1 - z_2 = z(t) = z_{10} e^{-\lambda t}\left(\cos\omega' t + \frac{\lambda}{\omega'}\sin\omega' t\right)$$

$$z_1 = \frac{z+z'}{2} = \frac{1}{2}\left[z_{10}\cos\omega t + z_{10} e^{-\lambda t}\left(\cos\omega' t + \frac{\lambda}{\omega'}\sin\omega' t\right)\right]$$

$$z_2 = \frac{z-z'}{2} = \frac{z_{10}}{2}\left[e^{-\lambda t}\left(\cos\omega' t + \frac{\lambda}{\omega'}\sin\omega' t\right) - \sin\omega t\right]$$

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